Randomized Methods for Network Security Games

João P. Hespanha

Joint work with: S. Bopardikar, M. Prandini, A. Borri, M. D. Di Benedetto

The Geography of Control

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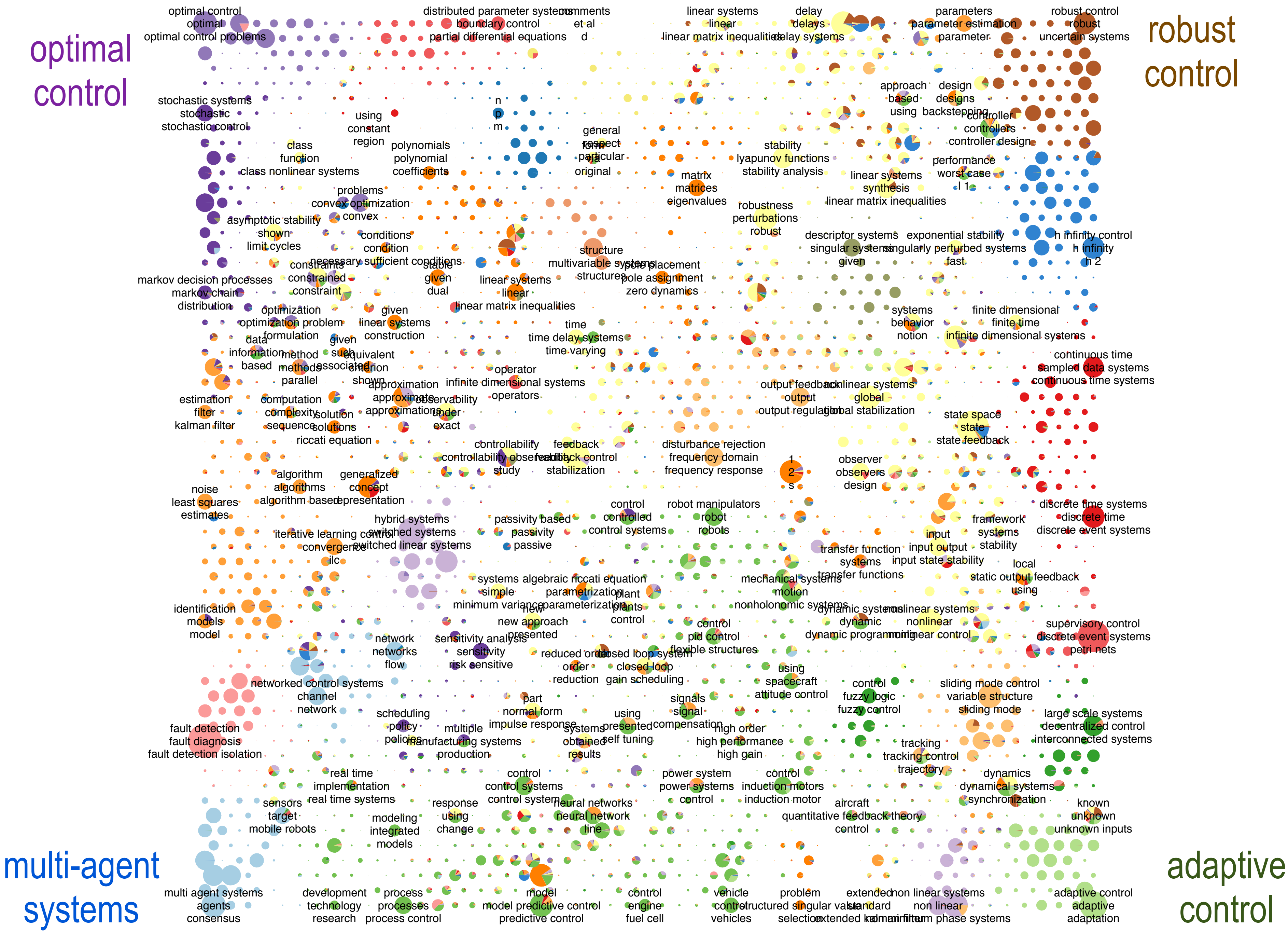

In collaboration with Stacy Rebich Hespanha

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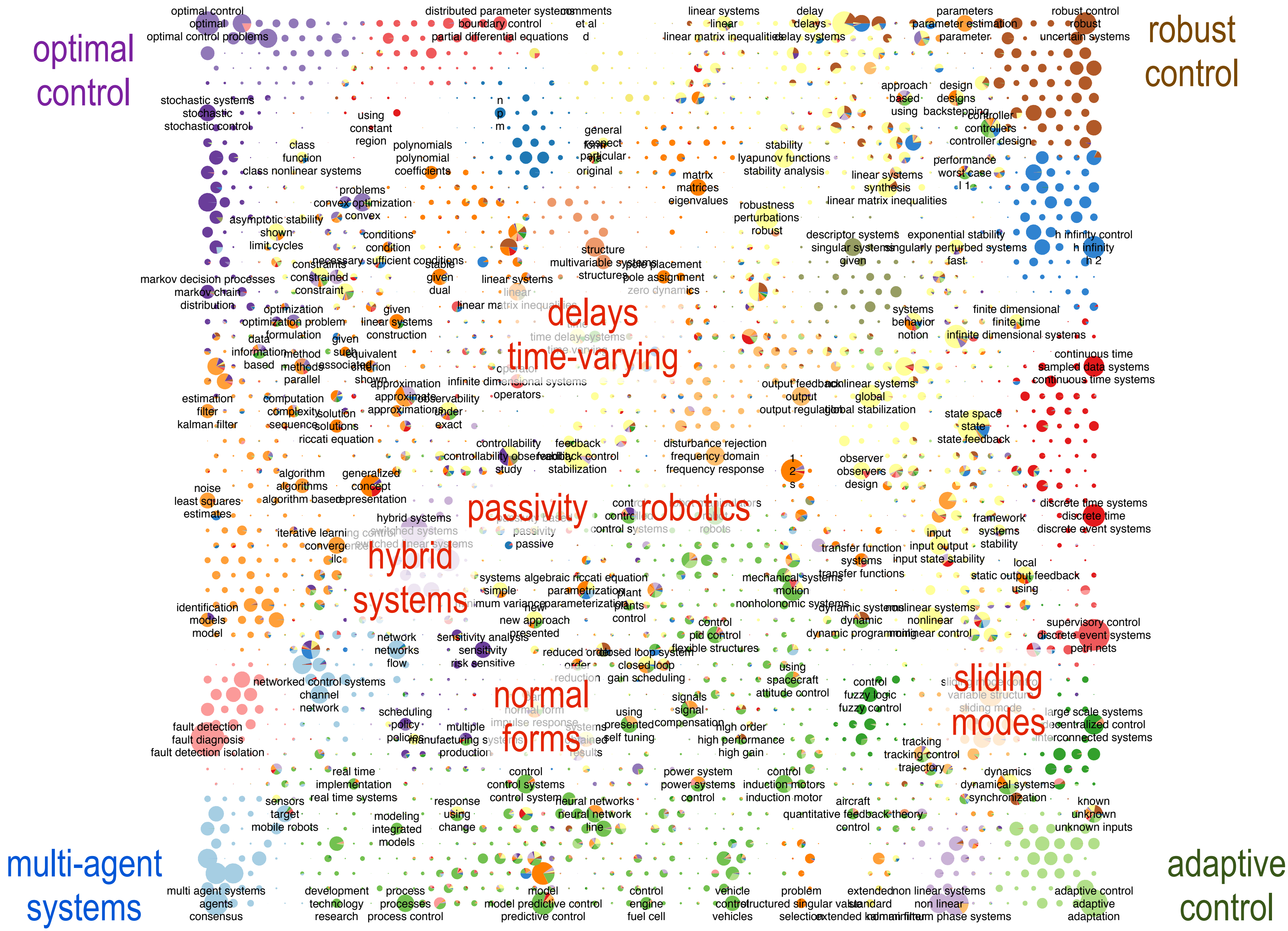

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The Geography of Mark Spong

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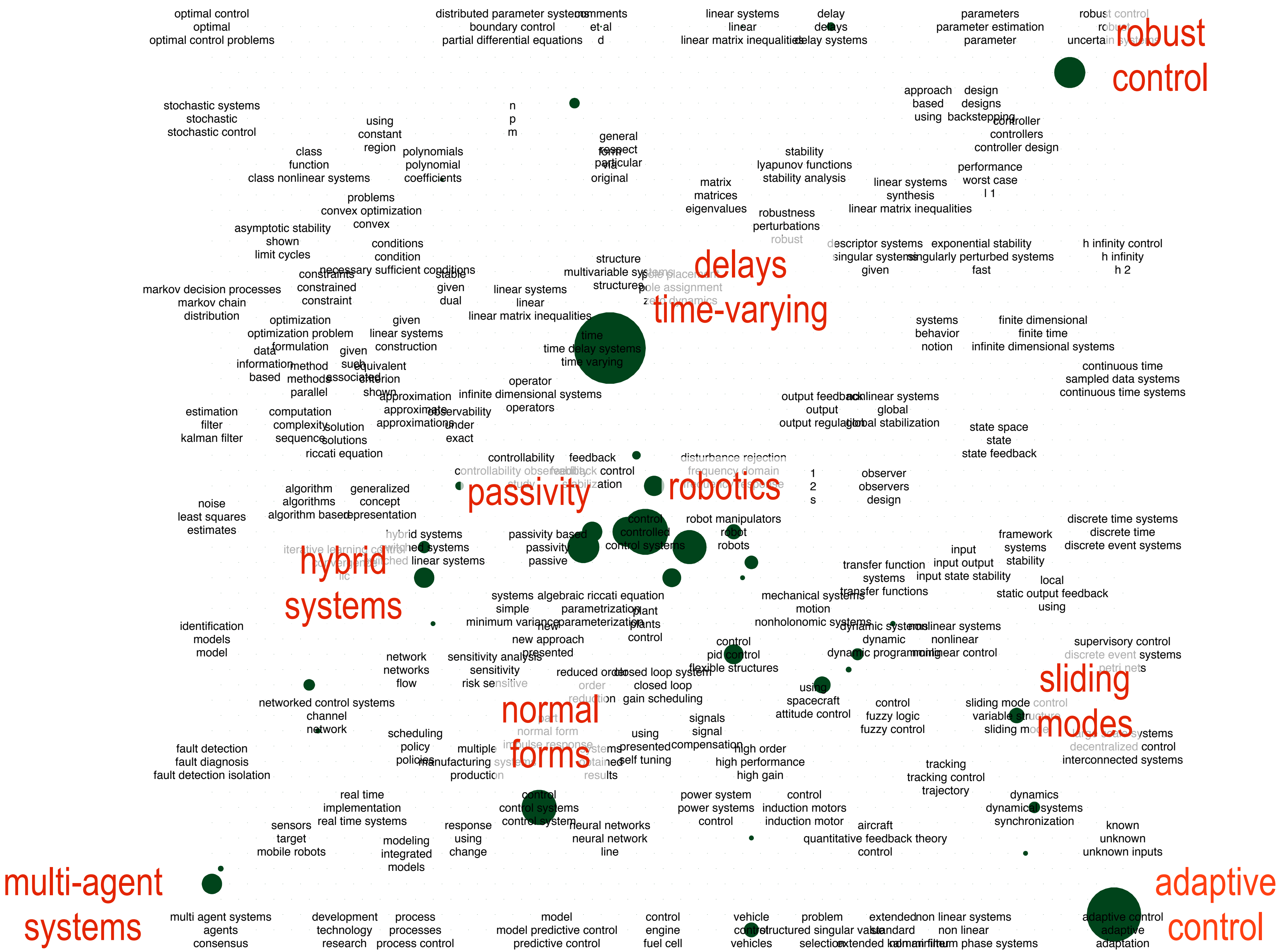

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The Plan...

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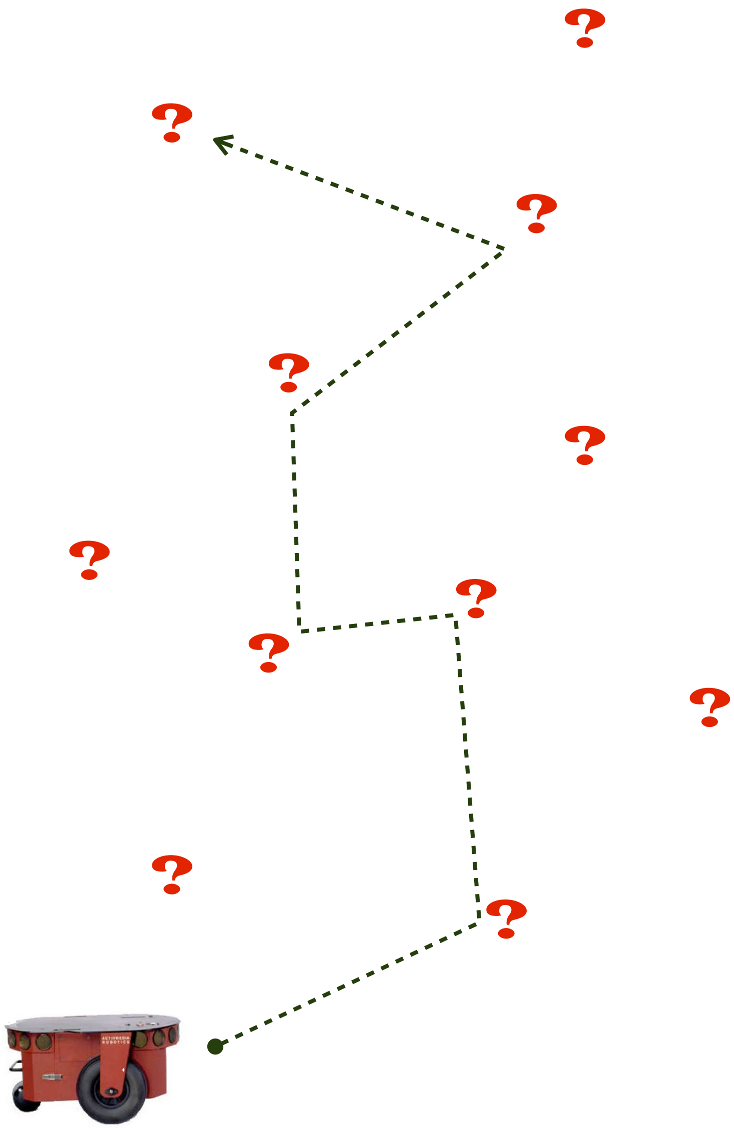
- Motivation for large-scale games
- Zero-Sum Games, Security Policies
- Randomized Algorithms for Games (SSP Algorithm)

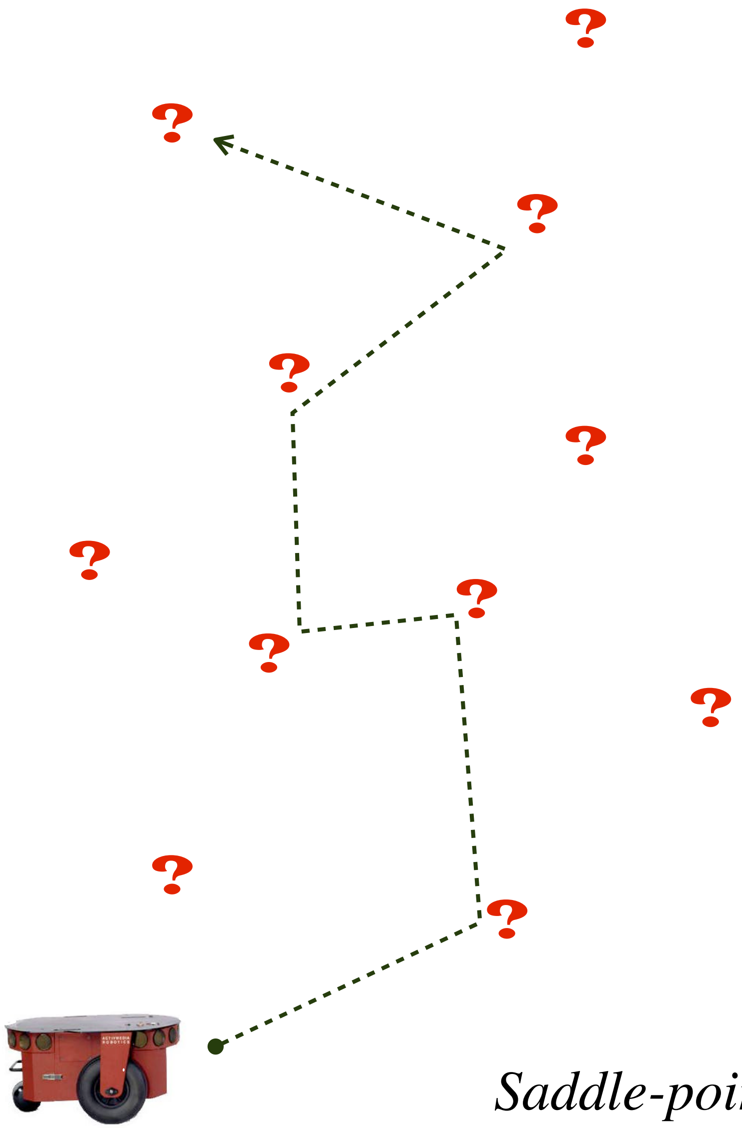
- Case Study in Network Security (time permiting...)

Path Planning Games: Hide & Seek



What is the "best" way to hide a treasure among N hiding places in the plane ?

Path Planning Games: Hide & Seek



What is the "best" way to hide a treasure among N hiding places in the plane ?

Formulate problem as *game*
- P1 selects hiding place (N locations)
- P2 selects robot path (**N! paths**)

Zero-sum game
- P1 wants to maximize time to reach treasure
- P2 has opposite objective

Saddle-point is a mixed policy

P1 places treasure based on "optimal" distribution
P2 selects path based on "optimal" distribution

Which???

Network Security

Cybaware

Cyber Situation Awareness ARO MURI:

Protecting the cyber infrastructure that supports "missions"

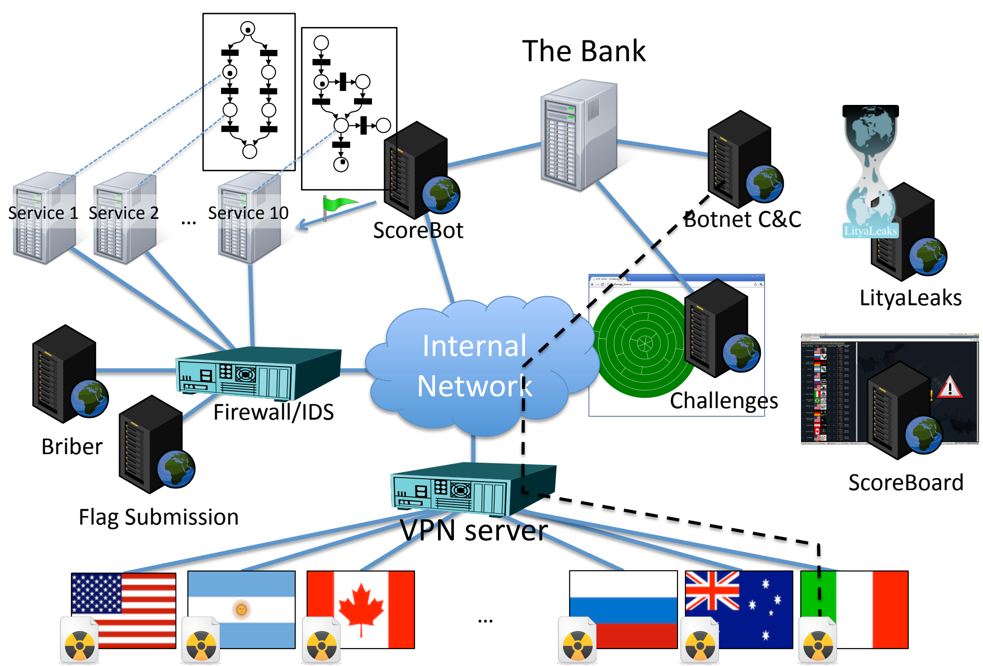
processing an online purchase

monitoring a critical infrastructure

managing the admissions process at a university

Cyber Awareness Questions:

- What is in the impact of a cyber-asset vulnerability (known or unknown)?
- What is in the impact of a particular counter measure (e.g., firewall control)?



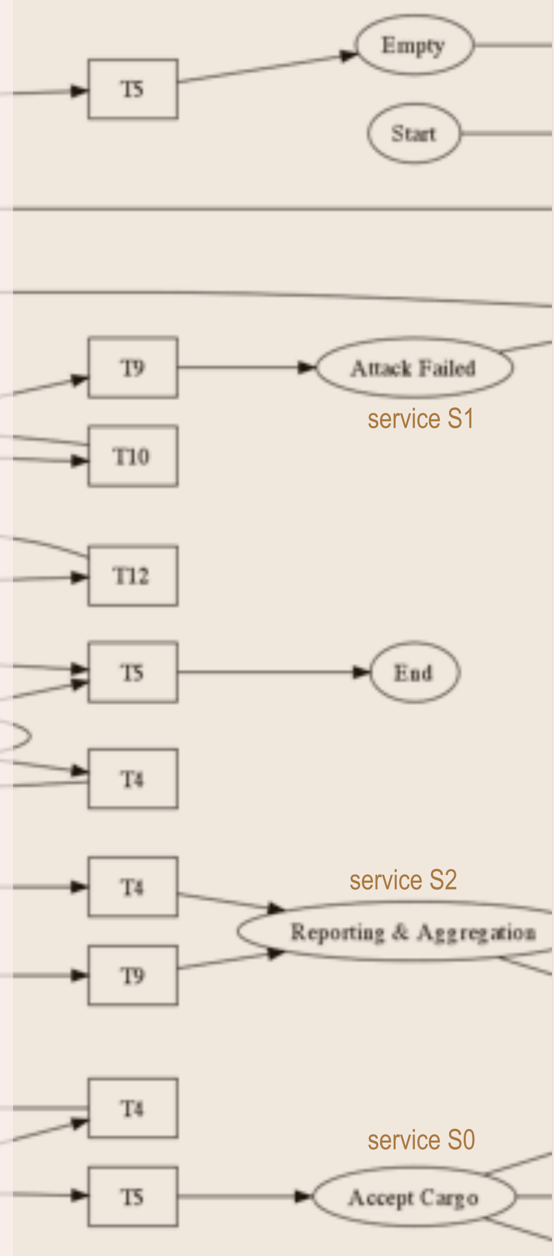
International Capture the Flag (iCTF):

- Distributed, wide-area security exercise to test the security skills of the participants
- Held yearly since 2003, under the organization of Prof. Giovanni Vigna @ UCSB
- 2010 edition involved 72 teams from around the world, over 900 participants

2010 iCTF Game

Key ingredients

1. *Game objective & Actors*
   - defender runs web server and collects revenue (when firewalls open) and bribes (when firewalls closed)
   - attacker collects revenue by compromising active services
2. *Players' Actions*
   - defender controls firewall
   - attacker explores service vulnerabilities
3. *Information Structure*
   - defender knows current status of several missions and services active
   - attacker has access to full state information (option I) or partial state information (option II)



Problem statistics:

- over 7800 distinct mission states (defender observations)
- over 2500 distinct observations available to the attacker
- defender can choose among about 10^{2527} distinct policies
- attacker can choose among 10^{756} – 10^{2616} distinct policies, depending on attacker's level of expertise

Randomized Search (H&S)

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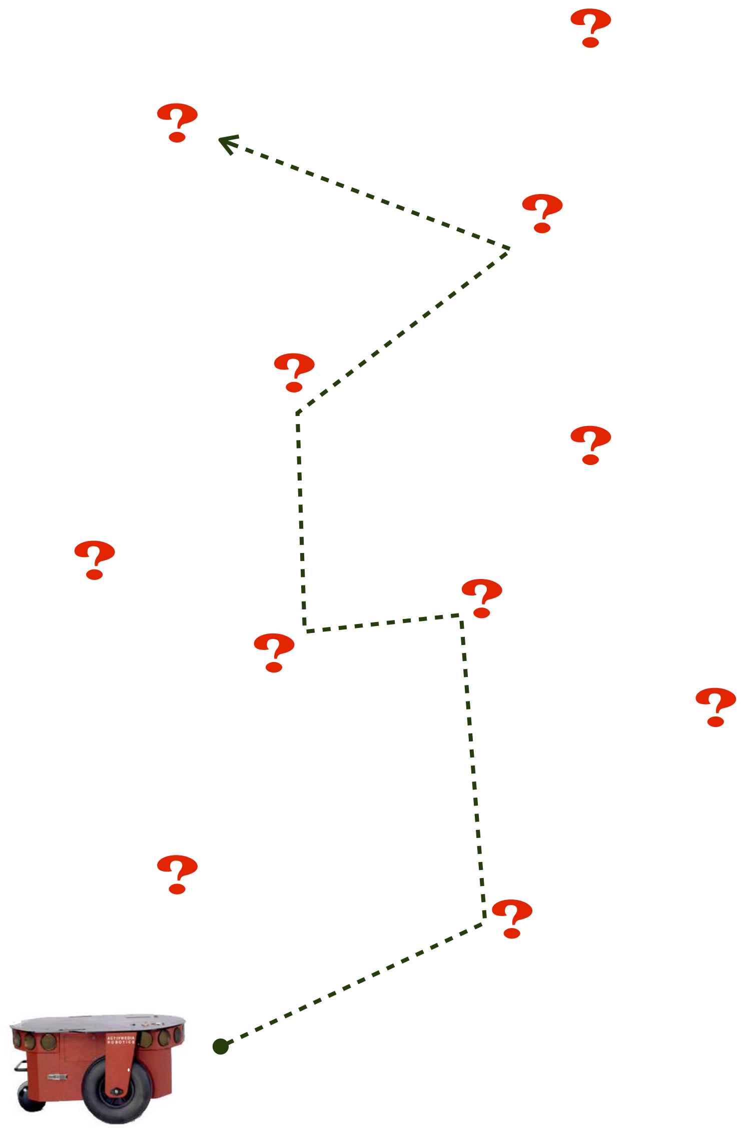


What is the "best" way to hide a treasure among N hiding places in the plane ?

- P1 selects robot path (**N! paths**)
- P2 selects hiding place (N locations)

Randomized algorithms:
When the exhaustive exploration of a combinatorial decision tree is not possible…
explore a random subset of it

Motivation for this work:
How can a player "protect" herself from an opponent engaged in random exploration?

Zero-Sum Matrix Games

Two players:
- P1 - minimizer
- P2 - maximizer

Each player selects a policy:
- S1 - set of available policies for P1
- S2 - set of available policies for P2

All possible game outcomes can be encoded in a matrix (2D-array), jointly indexed by the actions of the players

game outcome when
P1 selects policy i
P2 selects policy j

$$\begin{bmatrix} & \vdots & \\ \cdots & a_{ij} & \cdots \\ & \vdots & \end{bmatrix}$$

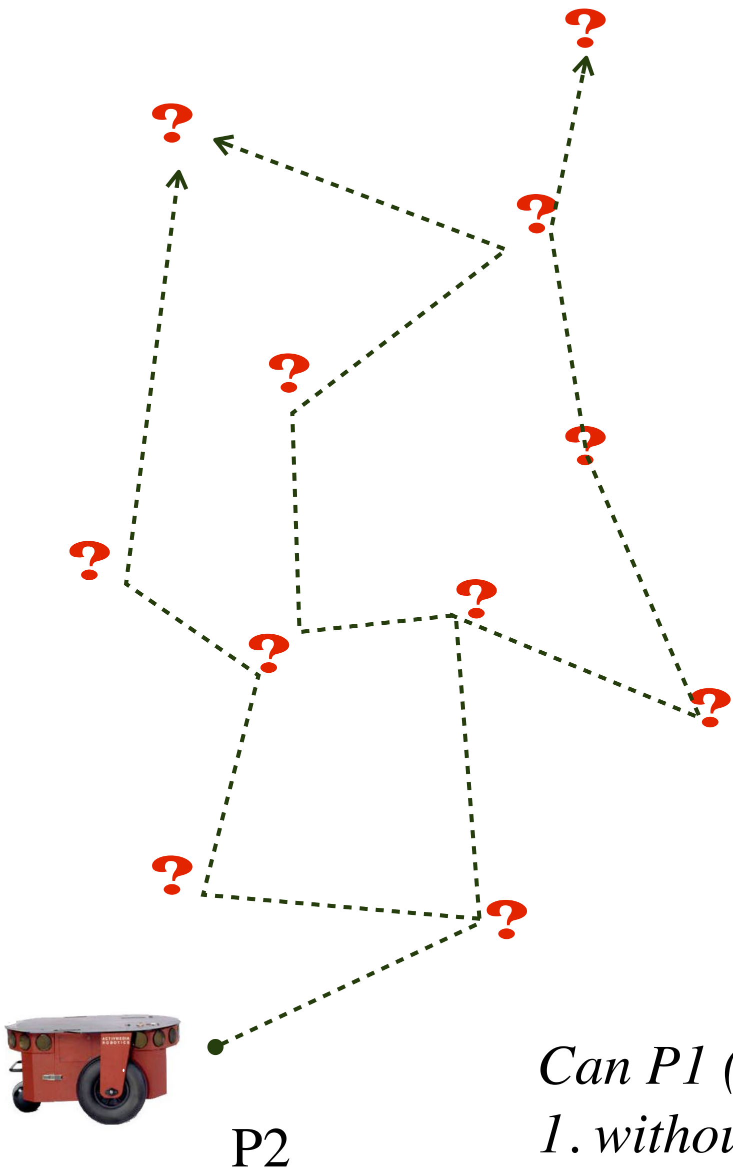
P1's policy

P2's policy

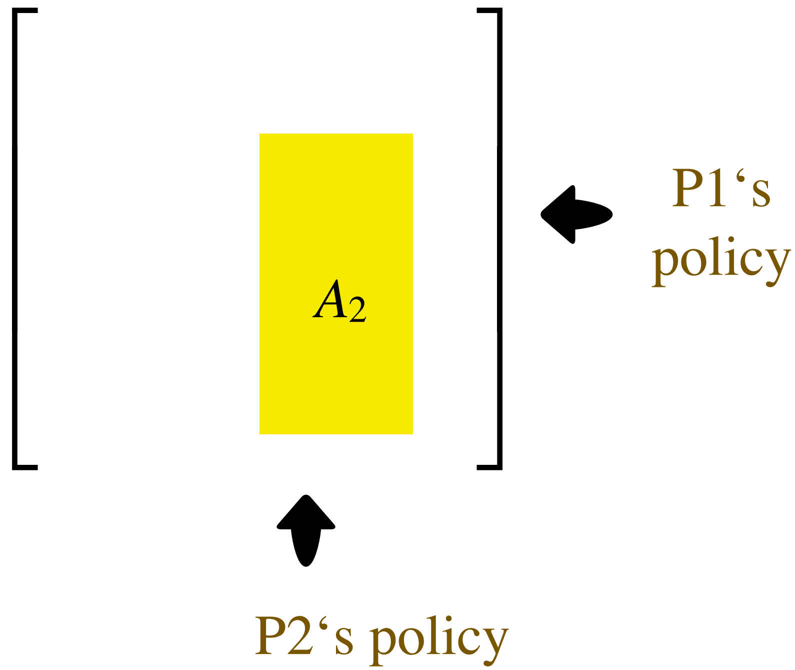
We are interested in games in which, at least one (possibly both) dimensions of this matrix are very large.

Mixed security level for P1 (minimizer): $V := \min_y \max_z \mathrm{E}[a_{ij}] = \min_y \max_z y' A z$

Randomized Search in Matrix Games



Suppose P2 (searcher) only considers an (unknown) random subset of the $N!$ paths and (somehow) selects a policy based on those...

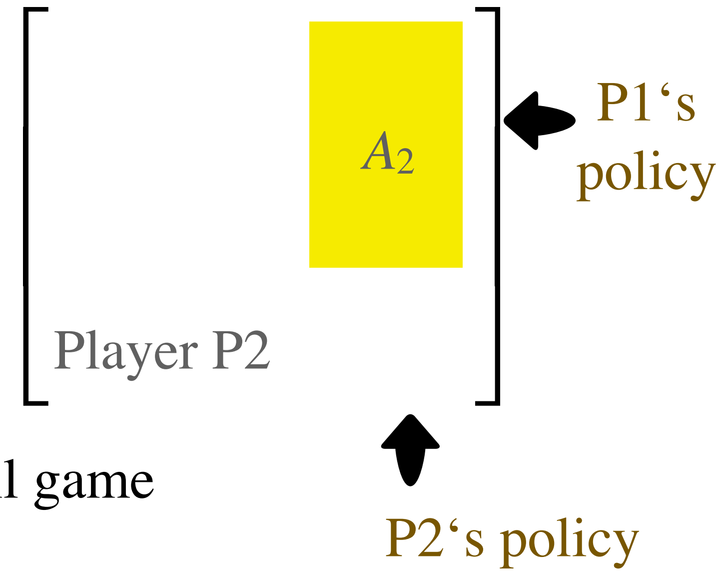


Can P1 (hider) guarantee some minimum search time

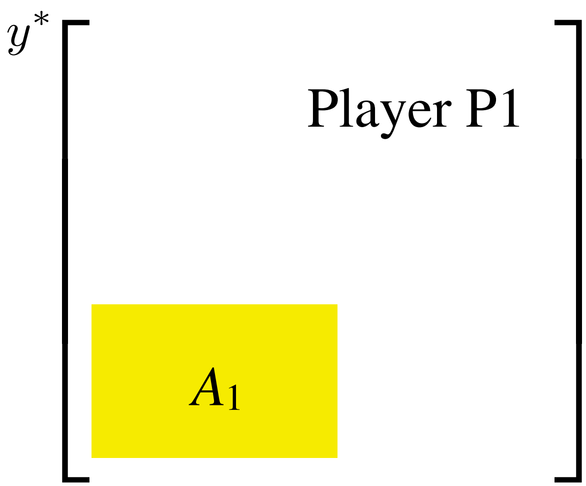
1. *without knowing which random subset was chosen*
2. *with a small computational effort ?*

Sampled-Security Policy Algorithm (SSP)

Suppose P2 only considers an (unknown) random subset of its policies to compute a (mixed) policy z^*...



1. Player P1 randomly selects a submatrix of the overall game



2. P1 solves the corresponding subgame (as if it was the whole game) and computes
   - mixed security level V_1
   - corresponding security policy y^*
3. P1 plays y^* against P2's policy z^*

in very large games, submatrix A_1 will likely not overlap the matrix A_2 used by P2

Sampled-Security Policy Algorithm (SSP)

Suppose P2 only considers an (unknown) random subset of its policies to compute a (mixed) policy z^*...

A_2

Player P2

1. Player P1 randomly selects a submatrix of the overall game

y^*

Player P1

A_1

in very large games, submatrix A_1 will likely not overlap the matrix A_2 used by P2

2. P1 solves the corresponding subgame
   - mixed security level V_1
   - corresponding security policy y^*
3. P1 plays y^* against P2's policy z^*

Because of independent subsampling, a player can now be unpleasantly surprised:

$$\underbrace{E_{y^*,z^*}[a_{ij}] > V_1}$$

outcome larger than minimizer expected based on its submatrix A_1

Security with High Probability

Probabilistic notion of security:

- probability of (unpleasant) surprises should be below a pre-specified bound
- with more computational power, one can demand lower prob. of surprise

Definition: The SSP algorithm is $\epsilon-$*secure for P1 (minimizer) with confidence 1*$-\delta$ if

$$\mathrm{P}(\underbrace{\mathrm{E}_{y*,z*}[a_{ij}] > V_1 + \epsilon}_{\substack{\text{outcome larger than} \\ \text{what P1 expected} \\ \text{(by more than } \epsilon\text{)}}}) \leqslant \delta$$

"Surprise" can arise because:

- our policy y^* is actually bad
- our security value V_1 is overly optimistic
- P2 was lucky in the selection of z^*

Security with High Probability

Probabilistic notion of security:

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avoidable with very high probability

not so easily under our control

Security with High Probability

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Definition: A particular policy y^* and value V_1 is ϵ−*secure for P1 (minimizer) with confidence 1*−δ if

$$\mathrm{P}(\mathrm{E}_{y*,z*}[a_{ij}] > V_1 + \epsilon \mid y^*, V_1) \leqslant \delta$$

"Surprise" only because:

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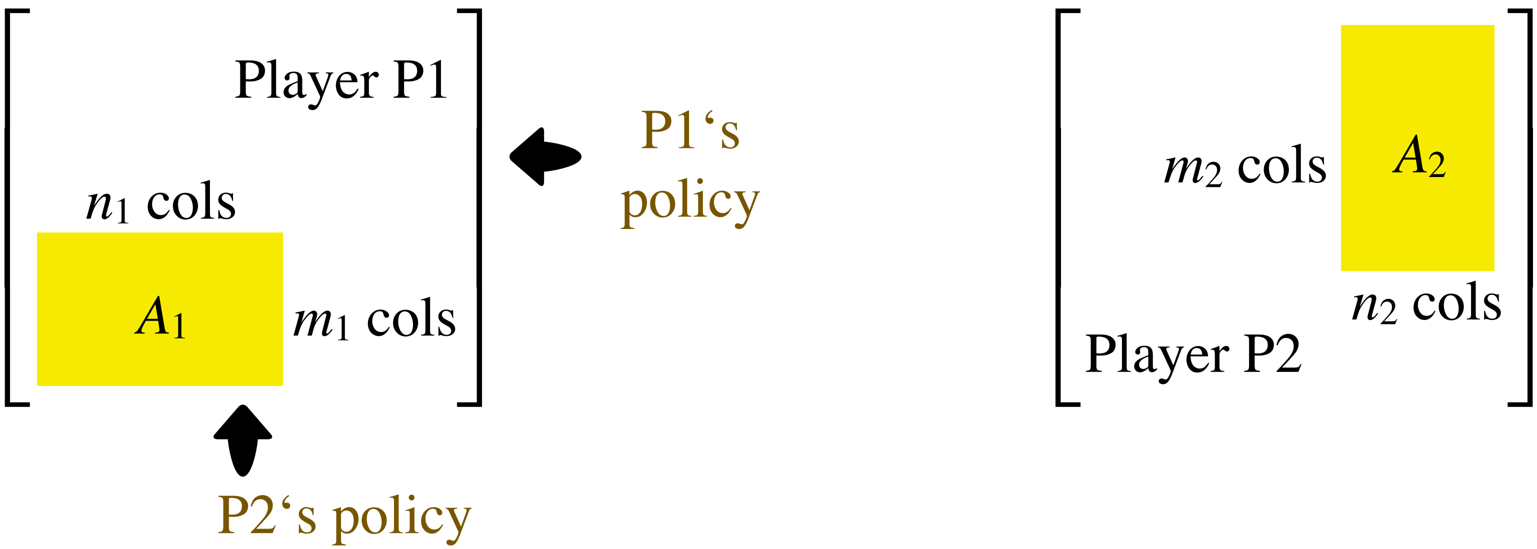
SSP Key Result



Theorem: The SSP algorithm is $\epsilon = 0$ – *secure for P1 (minimizer) with confidence 1–δ*, for

$$\delta = \frac{m_1 n_2}{n_1}$$

Conversely, to obtain desired confidence level δ, suffices to select

$$\frac{n_1}{m_1} \geqslant \frac{n_2}{\delta} > 1$$

"fat" sampling for A_1

$\Downarrow$

test more options for opponent than own (by appropriate ratio)

SSP Key Result

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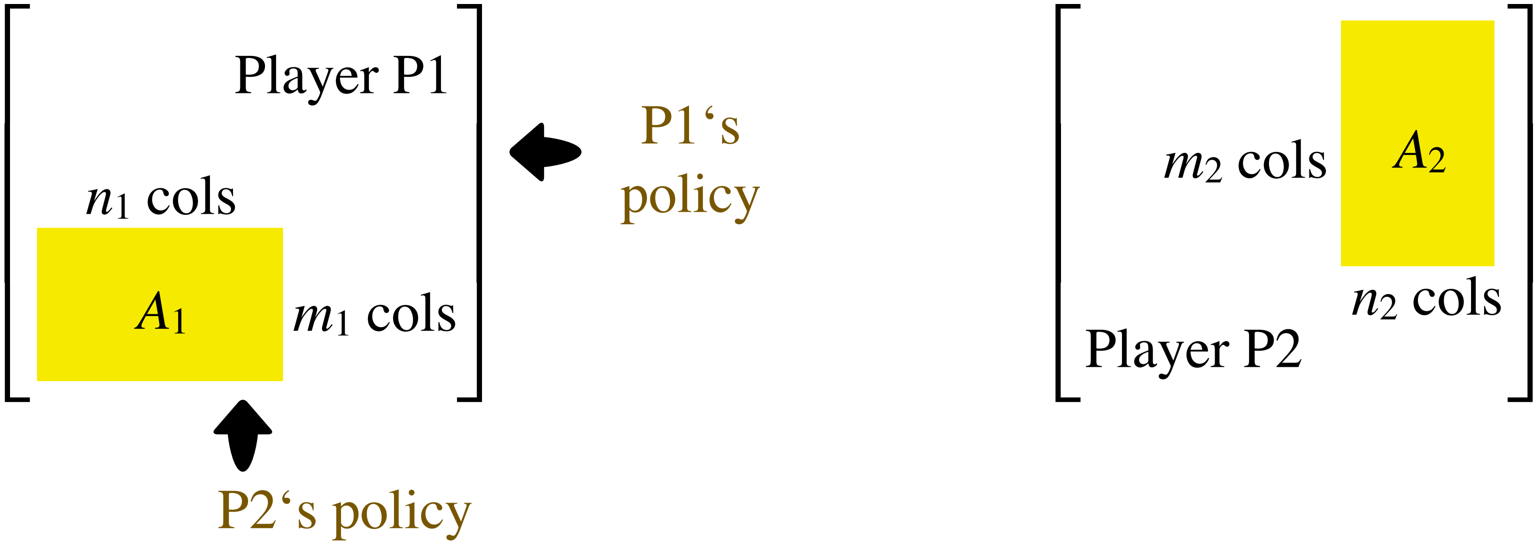

Theorem: The SSP algorithm is $\epsilon = 0$ – *secure for P1 (minimizer) with confidence* $1-\delta$, for $m_1 n_2$

Conversely, to obtain desired

$$\frac{n_1}{m_1} \geqslant \frac{n_2}{\delta} > 1$$

Game-independent bounds

- valid for any game
- independent of the size of the game

Bounds on *relative computation*

- required size of my sample depends on size of opponents' sample (n_2)
- the more I search for a good solution (large m_1), the more options need to consider for opponent (large n_1)

Security with High Probability (recall)

Definition: The SSP algorithm is $\epsilon-$*secure for P1 (minimizer) with confidence 1−δ* if

$$\mathrm{P}(\underbrace{\mathrm{E}_{y*,z*}[a_{ij}] > V_1 + \epsilon}_{\text{outcome larger than P1 expected (by more than } \epsilon\text{)}}) \leqslant \delta$$

"Surprise" can arise because:

- our policy y^* is actually bad
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Definition: A particular policy y^* and value V_1 is $\epsilon-$*secure for P1 (minimizer) with confidence 1−δ* if

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"Surprise" only because:

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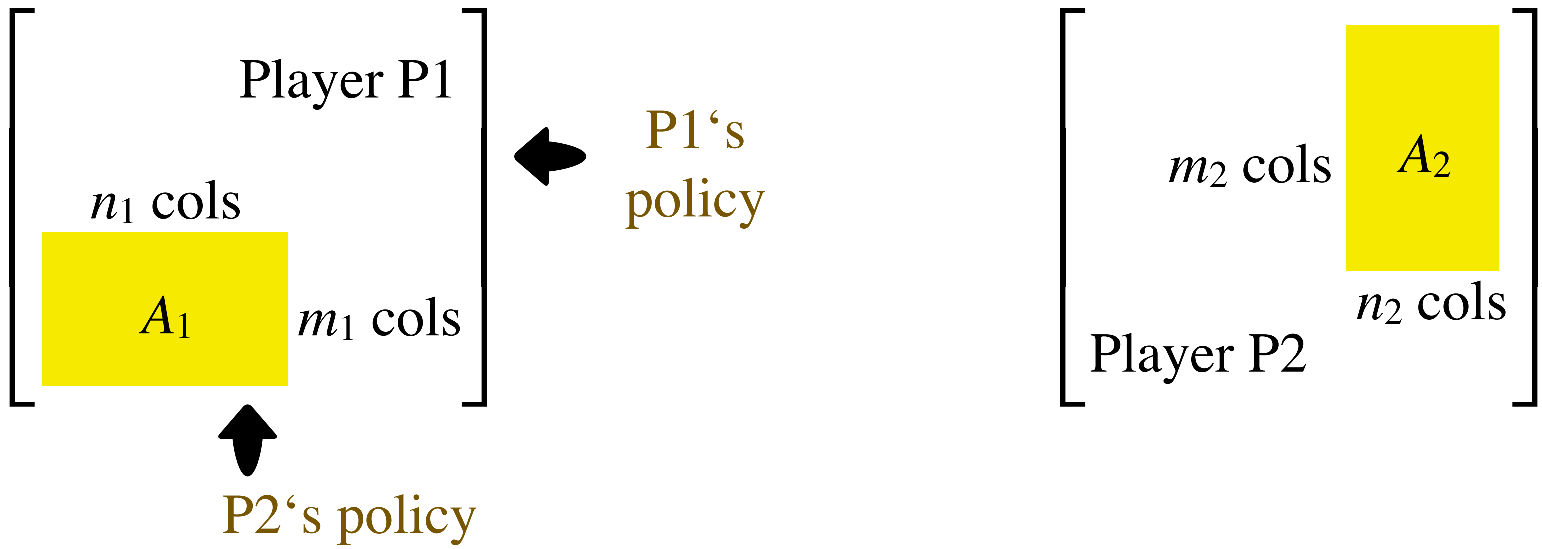
SSP Key Result (take 2)



Theorem: With probability larger than $1-\beta$, the SSP policy y^* and value V_1 are $\epsilon = 0$ – *secure for P1 (minimizer) with confidence* $1-\delta$, for

$$n_1 \geqslant \Big(m_1 + \underbrace{\sqrt{m_1 \ln \frac{1}{\beta^2}}}_{\text{additional term}}\Big)\frac{n_2}{\delta}$$

"Surprise" can arise because:

- our policy y^* is actually bad
- our security value V_1 is overly optimistic
- P2 was lucky in the selection of z^*

only with probability β (can be made extremely small ~ 10^{-9} with small computational cost) [for the first two items]

with probability δ [for the third item]

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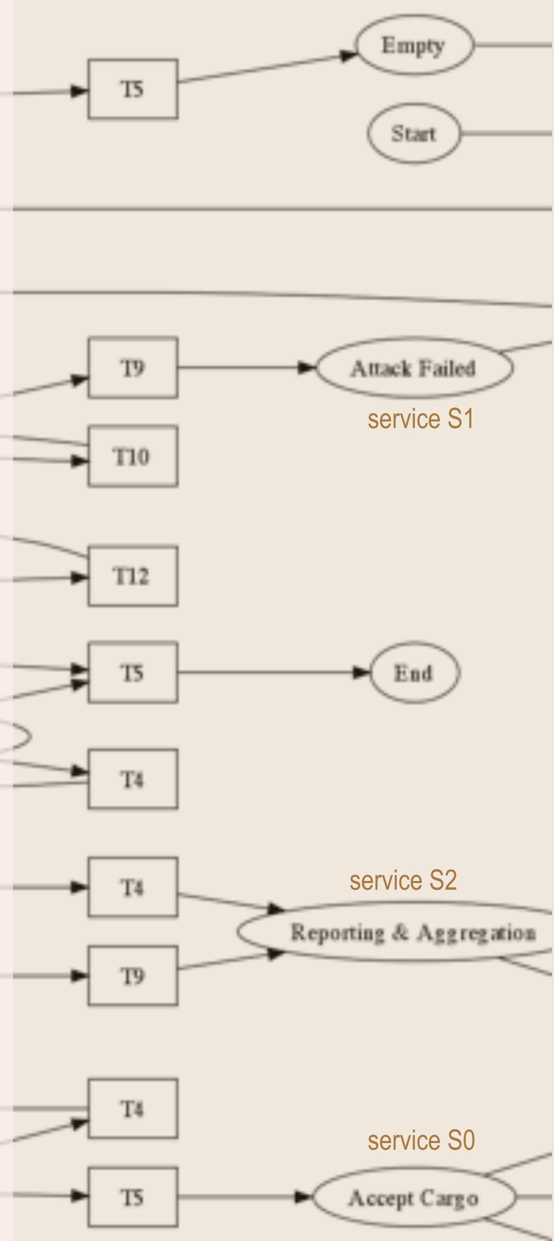

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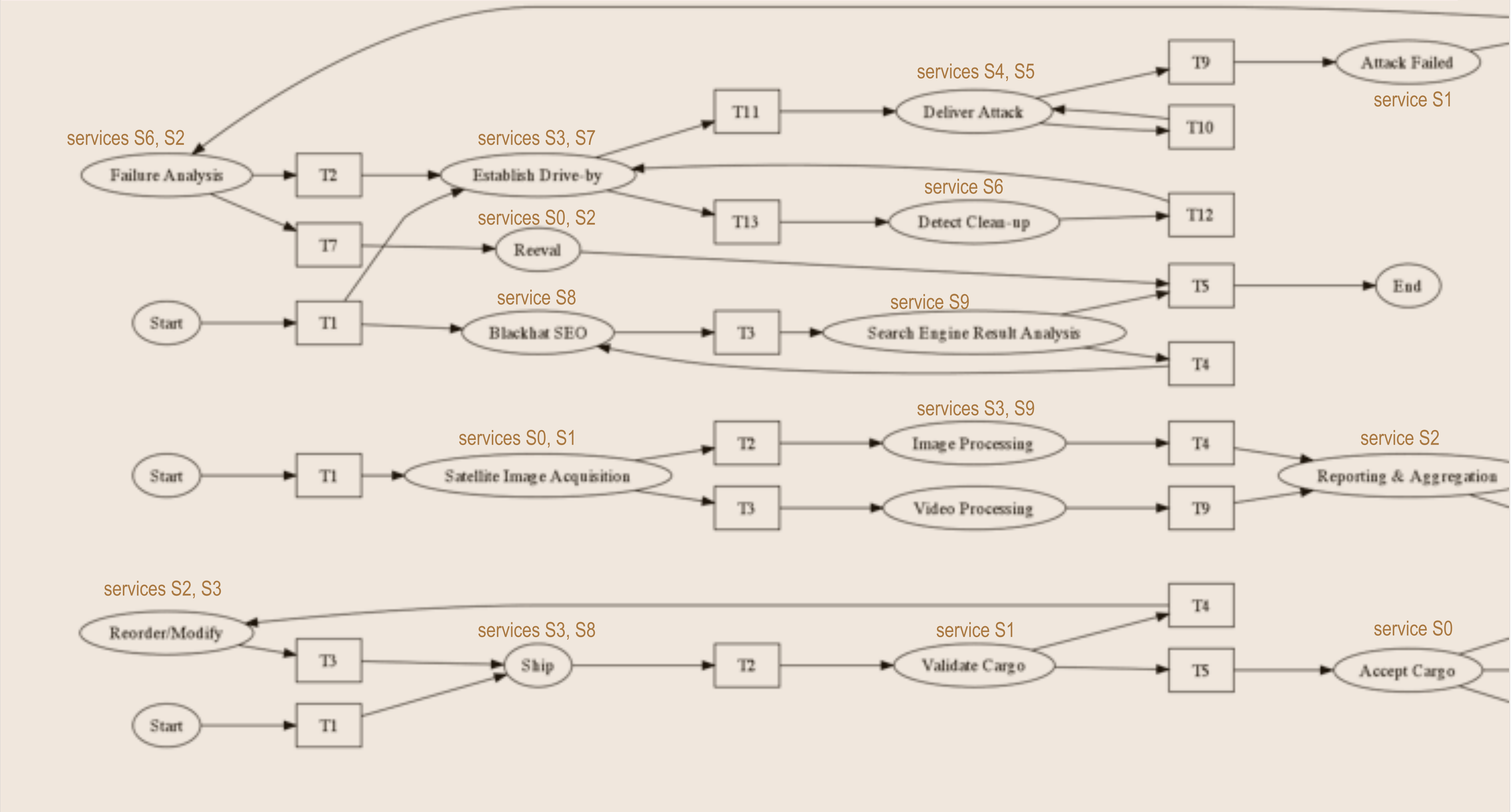
SSP-based Analysis

Baseline

- No attack
- Firewall always open
- All revenue from bot

Defender receives avg. 314 units for completion of 4 missions



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Attacks under Option I (ambiguous state information)

- Different levels of attacker sophistication
- Bribes not allowed

Increasing level of attacker sophistication

Level of attacker sophistication	# units received by Defender for 1 round of missions [Option I, no bribes]
no service vulnerable (baseline)	314
S2 (vulnerable to 38 teams)	240
S2, S6, S9 (vulnerable to at least 6 team)	79
S0, S2, S4, S6, S7, S8, S9 (vulnerable to at least 1 team)	11
all services vulnerable	11

Firewall closed whenever a vulnerable service is active

SSP-based Analysis

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Baseline

- No attack
- Firewall always open
- All revenue from bot

Defender receives avg. 314 units for completion of 4 missions

Attacks under Option II (full state information)

- Different levels of attacker sophistication
- Bribes allowed

Increasing level of attacker sophistication

Level of attacker sophistication	# units received by Litya for 1 round of missions [Option I, no bribes]	# units received by Litya for 1 round of missions [Option I, with bribes]	# units received by Litya for 1 round of missions [Option II, with bribes]
no service vulnerable (baseline)	314	314	314
S2 (vulnerable to 38 teams)	240	240	138
S2, S6, S9 (vulnerable to at least 6 team)	79	79	43
S0, S2, S4, S6, S7, S8, S9 (vulnerable to at least 1 team)	11	-738	-1327
all services vulnerable	11	-848	-1917

SSP-based Analysis

Baseline

- No a
- Fire
- All r

Attac

- Diff
- Bribes allowed

issions

- Provides Cyber-security office estimates of mission success
- Takes into account the effect of attacks & counter measures
- Responses as a function of attacker sophistication
- Ability to play what-if scenarios (vulnerabilities, information, etc.)

Increasing level of attacker sophistication

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Conclusions

Large matrix games are fun!

What I have covered:

- Basic probability guarantees of randomized sampling for games
- Case study in network security

What I have NOT covered:

- Can we determine the security level of an arbitrary policy obtained by a method other than SSP? Yes
- Mistmatch between the players' distributions
  - if we sample "better" than opponent, no change in results
  - if we sample "worse" than opponent, degradation in confidence level δ (but recoverable with more sampling)
- Can we improve upon these bounds? Unclear
- Multi-core parallel implementations
- Other applications...

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Thank You!